2. "General Methods in Analysis, for the resolution of Linear Equations in Finite Differences and Linear Differential Equations." By Charles James Hargreave, Esq., LL.B., F.R.S. &c.

The investigations presented in this paper consist of two parts; the first offers a solution, in a qualified sense, of the general linear equation in finite differences; and the second gives an analysis of the general linear differential equation with rational factors, so far as concerns its solution in series.

The author observes that there does not at present exist any general method of solving linear equations in finite differences of an order higher than the first; and that with reference to such equations of the first order, we obtain insufficient forms which are intelligible only when the independent variable is an integer. It is in this qualified sense that the solutions proposed in this paper are to be taken; so that the first part of these investigations may be considered as an extension of this form of solution from the general equation of the first order to the general equation of the nth order.

In the second part, the author points out a method by which the results of the process above indicated may be made to give solutions of those forms of linear differential equations whose factors do not contain irrational or transcendental functions of the independent variable, or contain them only in an expanded form.

This object is effected by means of the theorem, relative to the interchange of the symbols of operation and of quantity, propounded by the author in a former memoir published in the Philosophical Transactions (Part I. for 1848, p. 31). It is one of the properties of this singular analytical process that it instantaneously converts a linear equation in finite differences into a linear differential equation; so that whenever the former is soluble, the latter is soluble also, provided the result be interpretable; a condition satisfied when the functions employed are rational algebraical functions.

Notwithstanding the qualified character of the solutions previously obtained for linear equations in finite differences, the solutions obtained from them by this process are free from all restriction. The solutions in series can be written down at once from the equation itself, inasmuch as each series has its own independent scale or law of relation; and no difficulties arise from the appearance of equal or imaginary roots in the equation determining the incipient terms of the series. These circumstances do indeed cause a certain variation of form; but they do not compel us to resort to any special process in each individual case.

The perfect separation and independence of the scales, or laws of relation of the series enables the author to discuss the characters of the series with reference to their convergency or divergency, and to classify these equations into sets having peculiar and distinguishing properties in regard to this subject.

The first set includes those equations whose solutions can always be found in *convergent* series of *ascending* powers of the independent variable; and if in such case the equation be solved in series of *descending* powers (which can be done by this process), those series are certainly always *divergent.*

The distinguishing marks of this class of equations are,—that the factor of the highest differential coefficient contains one term only; and that (the terms being arranged in an ascending order) when this term is x^p, the factor of the next differential coefficient must not contain a term lower than x^{p-1}, the next not lower than x^{p-2}, and so on to the end.

The second set includes those equations whose solutions can always be found in *convergent* series of *descending* powers of the independent variable; and if in such case the equation be solved in series of *ascending* powers, they are always *divergent*.

The distinguishing marks of this class of equations are,—that the factor of the highest differential coefficient contains one term only; and that when this term is x^p, the next factor must stop at x^{p-1}, the next at x^{p-2}, and so on to the end.

The third set includes equations whose solutions can be found in series of *ascending* powers which for some values of the independent variable are *convergent*, and for other values *divergent*; and whose solutions can also be found in series of *descending* powers which are *divergent* for all values for which the other series are *convergent*, and *convergent* for all values for which the other series are *divergent*.

The distinguishing marks of this class of equations are,—that the factor of the highest differential coefficient contains two terms only, and that with reference to the first of such terms the equation is under the restriction mentioned with regard to the first set, and that with reference to the second of such terms it is under the restriction mentioned with regard to the second set.

The fourth set includes equations whose solutions are or may be *divergent* for some values of x, both in the ascending and descending series. In some cases, the ascending series is necessarily divergent, and the descending series convergent or divergent according to the value of x; in other cases, the descending series is necessarily divergent, and the ascending series convergent or divergent according to value; and in the remaining cases, both series are convergent or divergent according to value, but not so as to be necessarily complementary to each other in this respect.

The distinguishing marks of this class are,—that the first factor may contain more than two terms; and that *either* the restriction of the first set is transgressed with reference to the highest term, *or* the restriction of the second set is transgressed with reference to the lowest term. In this set the divergency arising from value is of a finite character; and, as the series approach without limit to ordinary recurring series, there is a probability that the passage from convergency to divergency is not attended with danger.

The fifth set includes equations whose solutions, whether in ascending or descending series, are always *necessarily divergent.*

The distinguishing mark of this class is, that it transgresses *both* the restrictions to one or other of which the last set is subjected. In this case the divergency is infinite, and appears to be of an unmanageable character.

The analogy of the process leads to a presumption, that in all

cases of divergency, above referred to, the corresponding convergent solutions are in series infinite in both the ascending and descending directions.

The author observes in conclusion, that the inverse calculus of the process here developed may be employed for the discovery of the generating functions of series whose laws of relation are given.

The Society then adjourned over the Easter holidays to meet again on the 19th of April.

April 19, 1849.

The EARL OF ROSSE, President, in the Chair.

A paper was read, entitled "On the Meteorology of the Lake District of Cumberland and Westmoreland." By John Fletcher Miller, Esq. Communicated by Lt.-Colonel Sabine, R.A., For. Sec. R.S., &c.

This paper contains the results of meteorological observations made during 1848, similar to those made in the same district in preceding years, which were last year communicated to the Society. On these results, the author remarks that the fall of rain in the lake district, during the year 1848, greatly exceeds the amount in any other year since the register was commenced in 1844; and that there is a similar excess with reference to the number of wet days. The total depth of rain, in 1848, at Seathwaite, the wettest station, was 160·89 inches; and of this quantity, 114·32 inches fell in the six months, February, July, August, October, November and December. In February there fell the unprecedented quantity 30·55 inches.

The mountains flanking the lake-district valleys increase in altitude with great regularity towards the head or eastern extremity of the vale, and it appears that it is there that the greatest depth of rain is invariably found. The amount increases rapidly as the stations recede from the sea, and towards the head of the valley the incremental ratio is exceedingly great. At Loweswater, Buttermere and Gatesgarth, about two miles apart in the same line of valley, the depths of rain were respectively 76 inches, 98 inches and 133·5 inches.

From the observations of the thermometer, the author concludes that the climate in the mountain valleys in this district is milder and more equable, not only than in the open country in their immediate vicinity, but also than in that considerably to the south. This he attributes to the lakes giving out during the winter the heat absorbed by them in the summer, and to the radiation from the rocky mountain breasts in the valleys, but principally to the heat evolved in a sensible form by the condensation of enormous volumes of vapour.

Last summer a pair of Rutherford's self-registering thermometers